

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/73
Paper 7 Probability \& Statistics 2 (S2)
May/June 2012
1 hour 15 minutes

Additional Materials: | Answer Booklet/Paper |
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| Graph Paper |
| List of Formulae (MF9) |

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Leaves from a certain type of tree have lengths that are distributed with standard deviation 3.2 cm . A random sample of 250 of these leaves is taken and the mean length of this sample is found to be 12.5 cm .
(i) Calculate a $99 \%$ confidence interval for the population mean length.
(ii) Write down the probability that the whole of a $99 \%$ confidence interval will lie below the population mean.

2 The independent random variables $X$ and $Y$ have the distributions $\mathrm{N}(6.5,14)$ and $\mathrm{N}(7.4,15)$ respectively. Find $\mathrm{P}(3 X-Y<20)$.

3 The lengths, $x \mathrm{~mm}$, of a random sample of 150 insects of a certain kind were found. The results are summarised by $\Sigma x=7520$ and $\Sigma x^{2}=413540$.
(i) Calculate unbiased estimates of the population mean and variance of the lengths of insects of this kind.
(ii) Using the values found in part (i), calculate an estimate of the probability that the mean length of a further random sample of 80 insects of this kind is greater than 53 mm .

4 The number of lions seen per day during a standard safari has the distribution $\operatorname{Po}(0.8)$. The number of lions seen per day during an off-road safari has the distribution $\operatorname{Po}(2.7)$. The two distributions are independent.
(i) Susan goes on a standard safari for one day. Find the probability that she sees at least 2 lions.
(ii) Deena goes on a standard safari for 3 days and then on an off-road safari for 2 days. Find the probability that she sees a total of fewer than 5 lions.
(iii) Khaled goes on a standard safari for $n$ days, where $n$ is an integer. He wants to ensure that his chance of not seeing any lions is less than $10 \%$. Find the smallest possible value of $n$.

5 (i) Deng wishes to test whether a certain coin is biased so that it is more likely to show Heads than Tails. He throws it 12 times. If it shows Heads more than 9 times, he will conclude that the coin is biased. Calculate the significance level of the test.
(ii) Deng throws another coin 100 times in order to test, at the $5 \%$ significance level, whether it is biased towards Heads. Find the rejection region for this test.

6 Last year Samir found that the time for his journey to work had mean 45.7 minutes and standard deviation 3.2 minutes. Samir wishes to test whether his journey times have increased this year. He notes the times, in minutes, for a random sample of 8 journeys this year with the following results.

$$
\begin{array}{llllllll}
46.2 & 41.7 & 49.2 & 47.1 & 47.2 & 48.4 & 53.7 & 45.5
\end{array}
$$

It may be assumed that the population of this year's journey times is normally distributed with standard deviation 3.2 minutes.
(i) State, with a reason, whether Samir should use a one-tail or a two-tail test.
(ii) Show that there is no evidence at the $5 \%$ significance level that Samir's mean journey time has increased.
(iii) State, with a reason, which one of the errors, Type I or Type II, might have been made in carrying out the test in part (ii).

7


A random variable $X$ has probability density function given by

$$
\mathrm{f}(x)= \begin{cases}k \sin x & 0 \leqslant x \leqslant \frac{2}{3} \pi \\ 0 & \text { otherwise }\end{cases}
$$

where $k$ is a constant, as shown in the diagram.
(i) Show that $k=\frac{2}{3}$.
(ii) Show that the median of $X$ is 1.32 , correct to 3 significant figures.
(iii) Find $\mathrm{E}(X)$.

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